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STABILITY OF A RECTANGULAR PLATE UNDER NONCONSERVATIVE AND CONSERVATIVE FORCES

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Abstract—A study is made of the flutter and divergence instabilities of a rectangular plate with two independent loading parameters. The plate is subjected to the combined action of a tangential follower force and a unidirectional axial force along one edge. Two opposite sides of the plate are simply supported, one side being clamped and the other being a free edge where the in-plane forces act. Depending on the relative magnitudes of the follower and axial forces, the plate may lose its stability by flutter or divergence. The flutter problem is solved by maximizing the flutter load over the frequency and thereby obtaining the maximum point of an eigencurve. The stability boundaries are given for plates with different aspect ratios.

The two-dimensional nature of the problem reveals some interesting results not observed in the one-dimensional counterpart of the problem, which is a cantilevered column under vertical and follower forces.

The effect of an elastic foundation on the stability boundaries are determined. The influence of Poisson's ratio on the flutter load and frequency is investigated. It is shown that no general rule can be formulated as to the effect of Poisson's ratio on the flutter load, and that this effect will vary according to the aspect ratio and axial load.

1. INTRODUCTION

Owing to its importance in many and diverse fields of technology and engineering, the elastic stability of nonconservatively loaded structures has been the subject of a large number of investigations, beginning with Beck's[1] now classical paper published in 1952.

The physical origins of nonconservative forces have been discussed by Herrmann[2], where various experimental studies of the subject were also reported. Reviews of the field are given in [3, 4] covering the periods before 1967 and 1975, respectively. Recent books by Huseyin[5] and Leipholz[6] provide up-to-date treatments of the elastic stability of nonconservative systems.

A characteristic feature of a structure under a follower load is the possibility that the structure may lose its stability owing to flutter. In such a case, a dynamic analysis of the problem is required to determine the critical flutter load. Mathematically, the dynamic analysis corresponds to the solution of an eigenvalue problem with double characteristic values which correspond to the flutter load and frequency in the physical problem.

Recently, attention has been devoted to the stability of structures subjected to the combined action of both nonconservative and conservative forces[2, 7–17]. Such a system can exhibit both flutter and divergence instability, depending on the relative magnitudes of the tangential and axial components of the forces acting on the system. These investigations revealed many interesting features of nonconservative stability problems, examined the transition of the system from divergence to flutter, and suggested possible ways of stabilizing a structure by the application of compressive or tensile follower forces.

The papers[2, 7–17] exclusively dealt with one-dimensional problems, among which special attention was devoted to the continuous case, i.e. that comprising beams, columns, and circular plates[2, 7–17]. In fact, studies of the stability of two-dimensional undamped structures have been rather scarce. Petterson[18] and Farshad[19] studied the stability of plates under subtangential follower loads. Leipholz[6, 20–22] studied a simply supported plate under a distributed tangential load, and a plate simply supported on three sides and free on the fourth, where a tangential edge load acts. In these two cases, the plates lose their stability only by divergence. Flutter instability was observed for clamped-free plates with two opposite edges simply supported and under a follower force acting at the free edge[23]. Using Liapunov's second method, Leipholz[24–26] investigated the asymptotic stability of damped plates.

In the present paper, we determine the stability regions of a rectangular plate subjected to a tangential follower force and a unidirectional axial force. In particular, we obtain the flutter and divergence boundaries of a plate simply supported along two opposite sides and clamped-free along the remaining sides, with the forces acting on the free edge. The numerical solution of the flutter problem is obtained by maximizing the flutter load over the frequency for a given axial load. This formulation permits the use of available function minimization routines and simplifies the solution of the characteristic equation. Previously, this problem was solved by computing the roots of a system of two nonlinear algebraic equations[1-23].

The numerical results indicate some interesting features of two-dimensional nonconservative systems. For example, for sufficiently high aspect ratios, the flutter load increases with increasing axial load after reaching a minimum.

The effect of an elastic foundation and the variation in Poisson's ratio on the flutter load and frequency are also studied. It is shown that the stability boundaries of a plate supported by an elastic foundation consists of a new divergence boundary and parts of the flutter boundary of the unsupported plate. Although Poisson's ratio has a considerable influence on the magnitude of the flutter load, no general rule can be formulated as to how this effect will vary with, say, increasing Poisson's ratio.

2. PROBLEM FORMULATION

We consider an isotropic, rectangular plate of length a, width b, thickness h, mass per unit area ρ, Poisson's ratio ν and Young's molulus E. The transverse displacement of its median surface is denoted by $W(X, Y, t)$, where X, Y are the Cartesian coordinates along two orthogonal sides, and t is the time (Fig. 1). We take the edges $Y = 0$ and $Y = b$ simply supported, the edge $X = 0$ clamped and the edge $X = a$ free. The free edge is subjected to a tangential follower force P_0 and an axial force N_0, both of which are uniformly distributed along the edge. The small transverse vibrations of the plate are governed by the differential equation

$$D\left(\frac{\partial^4 W}{\partial X^4}+2\frac{\partial^4 W}{\partial X^2 \partial Y^2}+\frac{\partial^4 W}{\partial Y^4}\right)+\rho\frac{\partial^2 W}{\partial t^2}+(P_0+N_0)\frac{\partial^2 W}{\partial X^2}=0 \tag{1}$$

subject to the boundary conditions

$$W=0, \quad \frac{\partial^2 W}{\partial Y^2}+\nu\frac{\partial^2 W}{\partial X^2}=0 \quad \text{at } Y=0, b \tag{2}$$

$$W=0, \quad \frac{\partial W}{\partial X}=0 \quad \text{at } X=0 \tag{3}$$

$$\frac{\partial^2 W}{\partial X^2}+\nu\frac{\partial^2 W}{\partial Y^2}=0, \quad D\left(\frac{\partial^3 W}{\partial X^3}+(2-\nu)\frac{\partial^3 W}{\partial X \partial Y^2}\right)+N_0\frac{\partial W}{\partial X}=0 \quad \text{at } X=a \tag{4}$$

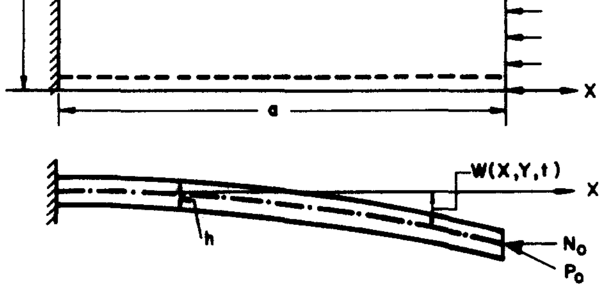

Fig. 1. The geometry and loading of the plate.

where $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate and the compressive edge forces are taken as positive.

Assuming a steady-state free vibration mode of the form

$$W(X, Y, t) = W_0(X, Y)\, e^{i\Omega t} \tag{5}$$

we separate the space and time coordinates in eqn (1). In eqn (5), Ω denotes the frequency of vibration. We introduce the following dimensionless quantities:

$$\begin{aligned} c &= a/b, \quad x = X/a, \quad y = Y/b, \quad w = W_0/a, \\ P &= P_0 a^2/D, \quad N = N_0 a^2/D, \quad \omega^2 = \rho\Omega^2 b^4/D. \end{aligned} \tag{6}$$

Substituting (5) and (6) into (1)–(4), we obtain the dimensionless governing equation

$$\frac{\partial^4 w}{\partial x^4} + 2c^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + c^4 \frac{\partial^4 w}{\partial y^4} - c^4\omega^2 w + (P+N)\frac{\partial^2 w}{\partial x^2} = 0 \tag{7}$$

subject to

$$w = 0, \qquad c^2 \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{at } y = 0, 1, \tag{8}$$

$$w = 0, \qquad \frac{\partial w}{\partial x} = 0 \quad \text{at } x = 0, \tag{9}$$

$$\frac{\partial^2 w}{\partial x^2} + \nu c^2 \frac{\partial^2 w}{\partial y^2} = 0, \qquad \frac{\partial^3 w}{\partial x^3} + c^2(2-\nu)\frac{\partial^3 w}{\partial x \partial y^2} + N\frac{\partial w}{\partial x} = 0 \quad \text{at } x = 1. \tag{10}$$

3. METHOD OF SOLUTION

A Levy-type solution can be applied to the present problem, viz.

$$w(x, y) = z(x)\sin m\pi y, \qquad m = 1, 2, \ldots \tag{11}$$

which satisfies the boundary conditions (8). Inserting (11) into (7), (9) and (10), we obtain

$$\mathscr{D}_x^4 z + (N + P - 2c^2m^2\pi^2)\mathscr{D}_x^2 z + c^4(m^4\pi^4 - \omega^2)z = 0 \tag{12}$$

subject to

$$z(0) = 0, \qquad \mathscr{D}_x z(0) = 0, \qquad \mathscr{D}_x^2 z(1) - \nu c^2 m^2\pi^2 z(1) = 0 \tag{13}$$

$$\mathscr{D}_x^3 z(1) + (N + c^2m^2\pi^2(\nu - 2))\mathscr{D}_x z(1) = 0 \tag{14}$$

where $\mathscr{D}_x = \mathrm{d}/\mathrm{d}x$. The solution of eqn (12) has the general form $z(x) = \mathrm{e}^{rx}$ where the complex number r is the root of the equation

$$r^4 + (N + P - 2c^2m^2\pi^2)r^2 + c^4(m^4\pi^4 - \omega^2) = 0. \tag{15}$$

From (15) it follows that

$$r^2 = \tfrac{1}{2}(-\delta_1 \pm (\delta_1^2 - 4\delta_2)^{1/2}) \tag{16}$$

where $\delta_1 = N + P - 2c^2m^2\pi^2$, $\delta_2 = c^4(m^4\pi^4 - \omega^2)$. Depending on the relative magnitudes of δ_1 and δ_2, the solution $z(x)$ of eqn (12) is given by different expressions.

Case I. $\delta_1^2 > 4\delta_2$

In this case, the roots r_i of (15) are either pure imaginary or real numbers. When $N + P >$

$2c^2m^2\pi^2$, we have

$$z(x) = c_1 \cos m_1x + c_2 \sin m_1x + c_3 \cos m_2x + c_4 \sin m_2x \tag{17}$$

if $m^4\pi^4 > \omega^2$ and

$$z(x) = c_1 \cos m_2x + c_2 \sin m_2x + c_3 e^{m_3x} + c_4 e^{-m_3x} \tag{18}$$

if $m^4\pi^4 < \omega^2$, where

$$m_1 = (\delta_1 - (\delta_1^2 - 4\delta_2)^{1/2})^{1/2}/2^{1/2}, \qquad m_2 = (\delta_1 + (\delta_1^2 - 4\delta_2)^{1/2})^{1/2}/2^{1/2}$$
$$m_3 = ((\delta_1^2 - 4\delta_2)^{1/2} - \delta_1)^{1/2}/2^{1/2}. \tag{19}$$

when $N + P < 2c^2m^2\pi^2$, we have

$$z(x) = c_1 e^{m_3x} + c_2 e^{-m_3x} + c_3 e^{m_4x} + c_4 e^{-m_4x} \tag{20}$$

if $m^4\pi^4 > \omega^2$ and $z(x)$ is given by (18) if $m^4\pi^4 < \omega^2$, where

$$m_4 = (-\delta_1 - (\delta_1^2 - 4\delta_2)^{1/2})^{1/2}/2^{1/2}. \tag{21}$$

Case II. $\delta_1^2 > 4\delta_2$

In this case, the roots r_i of (15) are complex numbers with nonzero real and imaginary parts and are given by $r_{1,2} = \pm((2\delta_2^{1/2} - \delta_1)^{1/2} + i(2\delta_2^{1/2} + \delta_1)^{1/2})/2$, $r_{3,4} = \bar{r}_{1,2}$, with bar denoting the complex conjugates. The solution is given by

$$z(x) = (c_1 e^{s_1x} + c_2 e^{-s_1x}) \cos s_2x + (c_3 e^{s_1x} + c_4 e^{-s_1x}) \sin s_2x \tag{22}$$

where

$$s_1 = (2\delta_2^{1/2} - \delta_1)^{1/2}/2, \qquad s_2 = (2\delta_2^{1/2} + \delta_1)^{1/2}/2.$$

By inserting the solution $z(x)$, determined by (17)–(22) depending on the values of the problem parameters, into (13) and (14), we obtain a linear system of four homogeneous algebraic equations in the unknowns c_i, $i = 1, \ldots, 4$. For a nontrivial solution, the determinant of this system should be equal to zero. The resulting nonlinear, transcendental equation

$$f(P, N, \omega; \nu, c, m) = 0 \tag{23}$$

is called the characteristic equation of the problem, the roots of which equation give the flutter and divergence loads P and N, and the flutter frequency ω for specified values of the problem parameters ν and c. Eqn (23) describes the characteristic hypersurfaces of the problem in the three-dimensional (P, N, ω)-space, among which hypersurfaces the one closest to the origin is called the fundamental characteristic surface. The projection on the subspace $\omega = 0$ of the (P, N, ω)-space of the fundamental characteristic surface provides the stability boundaries of the plate in the loading plane (P, N). The main objective of the present study is to determine these stability boundaries.

In the case of divergence instability, the critical values of P and N are computed by setting $\omega = 0$ in (23) and solving the resulting equation for P or N for a given value of N or P, respectively. Thus, a static stability analysis is sufficient to determine the divergence boundaries.

In the case of flutter instability, $\omega \neq 0$ is an unknown of the problem and should be computed together with P for a given value of N as the double eigenvalues of the problem (12)–(14). In this calculation, we make use of the well-known fact that two vibrational modes of the plate coalesce at the critical values of P and ω in the (P, ω)-plane[5] and (P, ω) are the coordinates of the maximum point of the corresponding eigencurve.

Customarily, this condition, i.e. $\partial P/\partial\omega = 0$ at the flutter load, is used to derive a second equation $\partial f/\partial\omega = 0$ from (23)[5]. Subsequently, the two equations are simultaneously solved for the unknowns P and ω. In our case, the complicated form of $f(P, N, \omega; \nu, c, m)$, due to the combinations of $z(x)$ given by (17)–(22) causes this method of computing to be somewhat cumbersome. Instead, we pose the problem as an optimization problem by noting that P reaches a maximum at the critical value of ω. This observation suggests a computational scheme by which P and ω are determined from the solution of the problem

$$\max_{\omega} P(\omega) \tag{24}$$

where for any given value of ω, P is the coordinate on the eigencurve and is found from (23).

Problem (24) can be solved by a nonlinear function maximization routine which does not require an explicit evaluation of the derivative $\partial f/\partial\omega$.

4. STABILITY BOUNDARIES

We determine the roots of the characteristic equation (23) by the method of bisection when two of the parameters P, N and ω are specified and the third is an unknown. The flutter load P and the corresponding frequency ω are computed by solving the problem (24) by using a quasi-Newton function maximization routine.

The lowest flutter loads P for all values of N are obtained when the first and second roots of ω coalesce when $m = 1$. There appears to be a typographical error in [23] where it is noted that 'the second and third roots approach each other' for this problem with $N = 0$.

On the other hand, the value given in [23] for the flutter load is 0.0192 when $N = 0$, $a/b = 1/3.0641$ and in our dimensionless variable it is $P = 12(a/h)^2 0.0192 = 23.04$ where $a/h = 10.0$. This value agrees very well with $P = 22.99$ obtained in our case.

The stability boundaries of the plate are shown in Fig. 2 for the aspect ratios $a/b = 0.5$, 1.0, 1.5 and $a/b = 0.0$, which corresponds to a clamped-free column. Throughout this section we

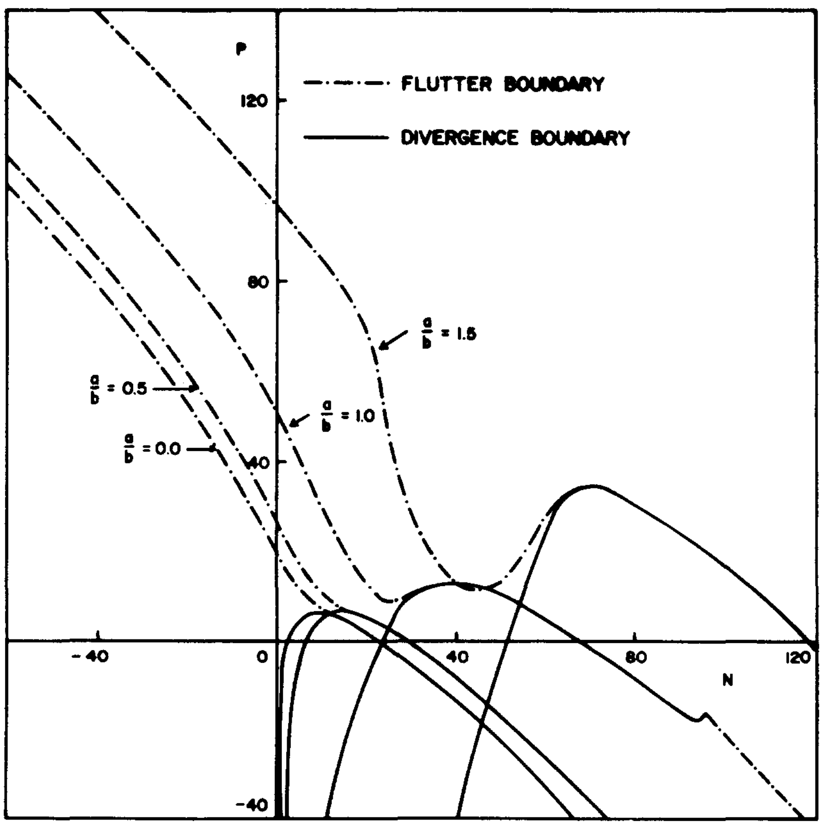

Fig. 2. Stability boundaries of the plates with aspect ratios $a/b = 0.0$, 0.5, 1.0, 1.5 and $\nu = 0.3$.

take $\nu = 0.3$. The results for the cantilever column are given here for comparison purposes and can also be found in [7, 16] where a characterization of follower and axial loads different from ours was used.

In the limit as $a/b \to 0$, the stability boundary of the plate converges to that of the column. This is a physically expected result, since the influence of the simply supported boundaries at $Y = 0$ and b diminishes as $a/b \to 0$.

The behaviour of the divergence boundary for cantilever columns was known in the case of concentrated loads[7, 16] and distributed loads[10, 12, 14]. A mechanical interpretation of the increase in stability when $P > 0$, and decrease when $P < 0$, was given in [10, 27]. This explanation seems to be valid also for two-dimensional structures.

The stability boundaries of the plates with $a/b \leq 0.5$ and $a/b \geq 1.0$ show a marked difference. The flutter boundary when $a/b \geq 1.0$ reaches a minimum point before contacting the divergence boundary. In other words, the flutter load decreases with increasing axial load N up to a certain point, and thereafter increases with increasing N up to the contact point, a phenomenon not observed in the case of columns with any combination of boundary conditions or loading distributions[7, 10–14, 16]. This is an intuitively unexpected result and it seems that as the effect of unloaded boundaries becomes more prominent, i.e. as a/b increases, the two-dimensional character of the problem becomes more effective and leads to the type of flutter boundaries shown in Fig. 2.

We remark that the second branch of the divergence boundary, which is of only theoretical importance, turns to a flutter boundary for sufficiently large N when $a/b = 1.0$.

Figure 2 corresponds to the loading plane (P, N) of the (P, N, ω)-space, and as such gives the boundary of the projection of the characteristic surfaces $f(P, N, \omega; \nu, c, m) = 0$ on the subspace $\omega = 0$. In the case of a divergence boundary, we always have $\omega = 0$, and consequently the characteristic surface emanating from this boundary is a cylinder in the (P, N, ω)-space.

Figure 3 shows the curves of P plotted against b/a for various values of N, together with the limiting values at $b/a = \infty$. The curve for $N = 0$ was given in [23].

5. PLATE ON AN ELASTIC FOUNDATION

Smith and Herrmann[28] have shown that a Winkler-type foundation has no effect on the critical flutter load of a cantilever column under a follower force. Subsequently, various aspects were investigated of nonconservatively loaded one-dimensional structures attached to an elastic

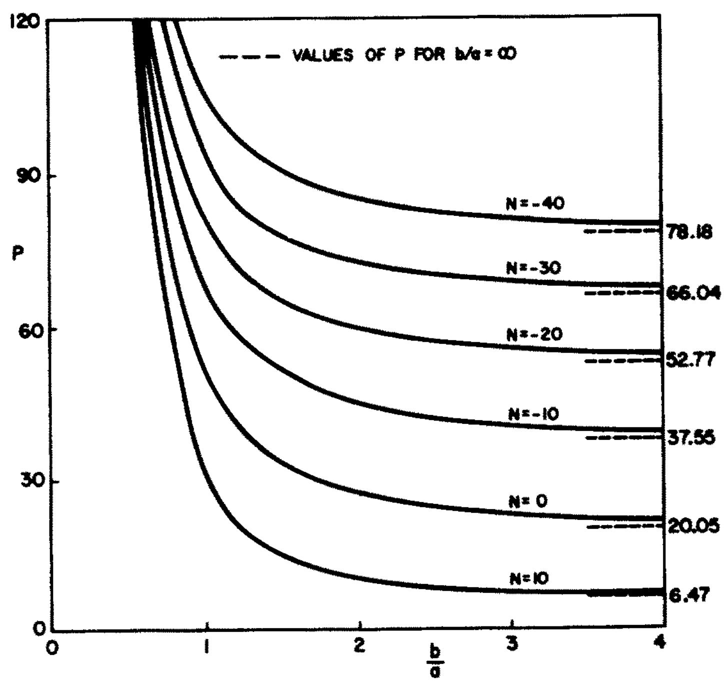

Fig. 3. Curves of the flutter load plotted against the ratio b/a with $\nu = 0.3$.

foundation[29–31]. Although an elastic foundation does not influence the magnitude of the flutter load and only causes a shift of the value of the flutter frequency, it changes the stability boundaries of the structure. In the present problem, the flutter boundary extends in the direction of increasing axial load and consequently the divergence boundary also changes.

We denote the elastic foundation modulus by K. With a solution of the form given by (11), the differential equation now is

$$\mathcal{D}_x^4 z + (N + P - 2c^2m^2\pi^2)\mathcal{D}_x^2 z + c^4(m^4\pi^4 + k - \omega_e^2)z = 0 \tag{25}$$

where the dimensionless quantity k is given by $k = b^4K/D$ and ω_e is the flutter frequency. The boundary conditions (13) and (14) remain the same.

Upon comparing (12) and (25), it becomes clear that the effect of the foundation is to increase the frequency parameter by an amount k, i.e. the new flutter frequency ω_e^2 is given by $\omega_e^2 = \omega^2 + k$. Consequently, at the contact point of the flutter and divergence boundaries where $\omega = 0$ for $k = 0$, $\omega_e^2 = k \neq 0$, and the flutter boundary extends with increasing N up to the point where $\omega_e^2 = 0$.

We illustrate these points in Fig. 4, where the stability boundaries in the first quadrant are given for a square plate with $\nu = 0.3$ and foundation moduli $k = 0, 50, 100, 150, 200$. We observe that the values of P and N at the contact point increase with increasing k. Part of the flutter boundary to the left of the point where $\omega = 0$, $k = 0$, remains the same for any $k > 0$, but the divergence boundary is different for each k.

6. EFFECT OF POISSON'S RATIO

In the mathematical formulation (11)–(16) of the problem, Poisson's ratio ν appears explicitly only in the boundary conditions (13) and (14) which apply at the loaded edge $x = 1$. However, the dimensionless parameters, P, N and ω implicitly depend upon ν through the term $D = Eh^3/12(1 - \nu^2)$ to which they relate by eqn (6). In order to observe the influence of ν, which

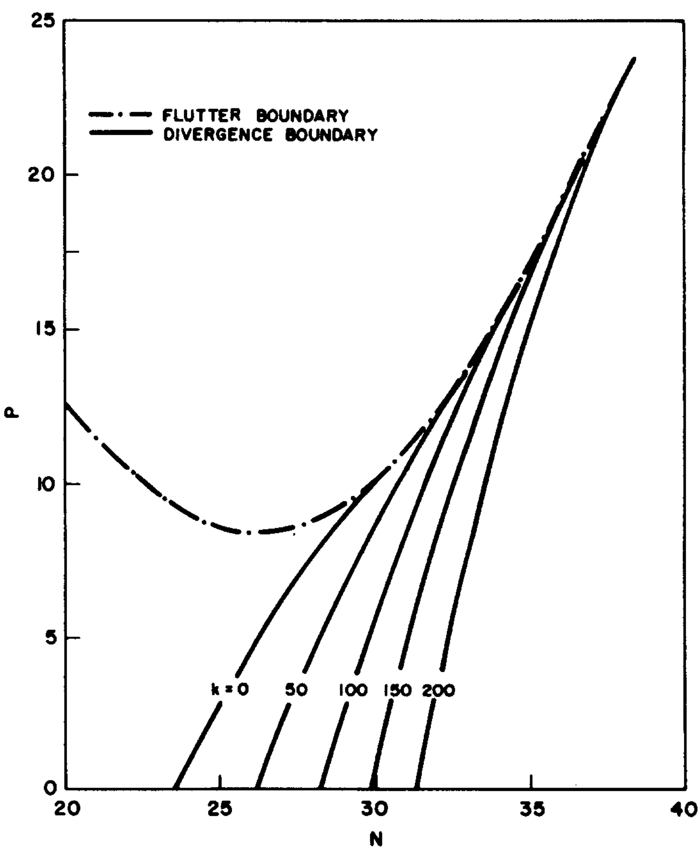

Fig. 4. Stability boundaries of a square plate resting on an elastic foundation of modulus k with $\nu = 0.3$.

can vary between 0.0 and 0.5 for isotropic materials, on load and frequency parameters which are independent of ν, we introduce the dimensionless quantities

$$\bar{P} = 12P_0a^2/Eh^3, \qquad \bar{N} = 12N_0a^2/Eh^3, \qquad \bar{\omega}^2 = 12\rho\Omega^2b^4/Eh^3. \tag{26}$$

From (6) and (26) it follows that

$$\bar{P} = P/(1-\nu^2), \qquad \bar{N} = N/(1-\nu^2), \qquad \bar{\omega} = \omega/(1-\nu^2)^{1/2}. \tag{27}$$

Clearly, the quantities P, N and ω depend on ν only through the boundary conditions at $x = 1$, whilst $\bar{P}$, $\bar{N}$ and $\bar{\omega}$ depend on ν through the differential equation (12) owing to the substitution of (27) into (12), and through the boundary conditions at $x = 1$.

Table 1 gives the values of P and ω for various values of N with $\nu = 0.0$, 0.3, 0.5 and $a/b = 0.5$, 1.0. We observe that P increases with increasing ν for $N = -20$, $a/b = 0.5$ and for $N = -20$, -10, $a/b = 1.0$, and decreases for higher values of N. The flutter frequency ω increases with increasing ν for $N = -20$, -10, 0 but decreases for $N = 10$ when $a/b = 0.5$. The behaviour of ω is more complicated when $a/b = 1$, in which case it shows a non-monotonic behaviour for $N = -20$, -10, increases for $N = 0$, 10, and decreases for $N = 20$ with increasing ν.

Leissa[32] notes that the effect of ν on the free vibration frequencies of plates was investigated for only some of the boundary conditions and that only one mode of the plate with all edges free has increasing frequency parameter ω with increasing ν. In the present problem, the variation of P and ω with ν depends largely on the magnitudes of N and a/b, and no general conclusions can be drawn.

Table 2 gives the values of $\bar{P}$ and $\bar{\omega}$ for various values of $\bar{N}$ with $\nu = 0.0$, 0.3, 0.5 and $a/b = 0.5$, 1.0. We observe that the variation with ν of $\bar{P}$ and $\bar{\omega}$, which reflect the actual magnitudes of the follower load and the flutter frequency, is more uniform as compared with

Table 1. The effect of Poisson's ratio ν on the flutter load P and frequency ω

	a/b = 0.5			a/b = 1.0		
ν	0.0	0.3	0.5	0.0	0.3	0.5
			N = -20			
P	59.50	59.67	59.82	77.87	81.33	85.75
ω	48.78	49.97	50.70	15.28	15.47	14.89
			N = -10			
P	45.37	44.78	44.32	65.71	68.01	71.86
ω	49.22	50.90	52.01	15.43	15.99	15.76
			N = 0			
P	28.87	27.11	25.80	52.40	51.65	50.57
ω	47.96	49.58	50.52	15.55	16.67	17.44
			N =10			
P	11.71	10.30	9.60	37.12	30.65	24.14
ω	38.19	36.65	34.92	15.53	16.91	17.36
			N =20			
P	-	-	-	19.59	12.59	10.99
ω	-	-	-	14.75	14.36	12.59

Table 2. The effect of Poisson's ratio ν on the flutter load $\bar{P}$ and frequency $\bar{\omega}$

	a/b = 0.5			a/b = 1.0		
ν	0.0	0.3	0.5	0.0	0.3	0.5
			$\bar{N}$ = -20			
$\bar{P}$	59.50	62.82	69.91	77.87	86.88	105.62
$\bar{\omega}$	48.78	52.60	59.40	15.28	16.30	17.62
			$\bar{N}$ = -10			
$\bar{P}$	45.37	47.59	53.26	65.71	73.29	90.17
$\bar{\omega}$	49.22	53.39	60.18	15.43	16.82	18.58
			$\bar{N}$ = 0			
$\bar{P}$	28.87	29.79	34.41	52.40	56.76	67.43
$\bar{\omega}$	47.69	51.97	58.34	15.55	17.48	20.14
			$\bar{N}$ = 10			
$\bar{P}$	11.71	12.60	16.95	37.12	35.85	39.74
$\bar{\omega}$	38.19	40.81	47.99	15.53	17.78	20.55
			$\bar{N}$ = 20			
$\bar{P}$	-	-	-	19.59	16.51	20.78
$\bar{\omega}$	-	-	-	14.75	15.88	18.05

that of P and ω. $\bar{P}$ and $\bar{\omega}$ increase with increasing ν for all values of $\bar{N}$ and a/b, excpet that $\bar{P}$ first decreases and then increases when $\bar{N} = 10$, 20 and $a/b = 1.0$.

That the frequency increases with increasing ν could reasonably have been expected, but it is not possible to make a general statement about the effect of ν on the flutter load P. We note that the rate of increase of $\bar{P}$ and $\bar{\omega}$ itself increases as ν approaches the upper limit $\nu = 0.5$.

7. CONCLUDING REMARKS

By exact analysis, a study has been made of the loss of stability, by flutter and divergence, of a rectangular plate subjected to nonconservative and conservative edge loads.

The effect of an elastic foundation on the stability boundaries was determined. The effect of Poisson's ratio on the flutter load and frequency was examined for various axial loads and aspect ratios.

It was found that the aspect ratio has a strong qualitative and quantitative influence on the stability boundaries. For example, for sufficiently large aspect ratios, the flutter load increases with increasing axial load after having reached a minimum value, i.e. the flutter load has a minimum point with respect to the conservative load.

This rather unexpected phenomenon is difficult to explain intuitively and does not seem to apply to one-dimensional structures [2, 5–17]. This observation has implications in determining the combination of follower and unidirectional axial forces which may stabilize or destabilize a two-dimensional structure.

The effect of an elastic foundation on the stability boundaries was studied and these boundaries are determined for a square plate for various values of the foundation moduli. Also investigated was the influence of Poisson's ratio $0 \leq \nu \leq 0.5$ on the flutter load and the frequency. It was found that the only general conclusion is the increase in the flutter frequency of the plate with increasing ν. The flutter load may increase, decrease or become non-monotonic with increasing ν, depending on the magnitude of the axial load and/or the aspect ratio.

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